पूर्वोत्तर पर्वतीय विश्वविद्यालय
जिला: पूर्वी खासी हिल्स, शिलांग - 793022
मेघालय (भारत)

North-Eastern Hill University
District: East Khasi Hills, Shillong - 793022
Meghalaya (India)

No. ACAD/11/2161 Date: 01-11-2024

NOTICE

There will be an Entrance test for admission into the Ph.D. programme in Mathematics on the 10th December, 2024 from 10.30 a.m. to 1.30 p.m. in Mathematics Department, NEHU Shillong. The list of eligible candidates for the entrance test and a model question paper are appended below.

Only those who score minimum 50% marks in the written test (5% concession for SC/ST/OBC (Non-creamy layer)) will be called for an interview to be held on 11th December, 2024 in the Conference Room of the Department. The time of the interview will be notified later.

01-11-2024

Head, Department of Mathematics
NEHU, Shillong.

Head
Department of Mathematics
North-Eastern Hill University
Shillong-793022

2024
(November)

MATHEMATICS

(Ph.D. Admission Test - Model Questions)

Full Marks : 100
Time : 3 hours

Answer **25** questions from **Group A** and choose any **05** from **Group B**. All the symbols have their usual meaning unless otherwise specified.

Group A

1. One may split the overall research design into _____ number of parts.
   (a) one
   (b) two
   (c) three
   (d) four

2. The attribute 'intelligence' of a person is treated as a/an _____
   (a) extraneous variable.
   (b) independent variable.
   (c) dependent variable.
   (d) erroneous variable.

3. The research hypothesis is a predictive statement that relates a/an _____
   (a) independent variable to a extraneous variable.
   (b) dependent variable to an erroneous variable.
   (c) dependent variable to a extraneous variable.
   (d) independent variable to a dependent variable.

4. The attribute 'age' of a person is considered as a/an _____ variable.
   (a) extraneous
   (b) erroneous

(c) dependent

(d) independent

5. The different conditions under which experimental and control groups are put in research design are usually referred to as _____

   (a) treatments.

   (b) constraints.

   (c) quickings.

   (d) obstincts.

6. The pre-determined plots, where different treatments are used in research design are known as _____

   (a) experimental bugs.

   (b) experimental mappings.

   (c) experimental polls.

   (d) experimental units.

7. The number of principles enumerated by Prof. Fisher of an experimental design are _____

   (a) two.

   (b) four.

   (c) three.

   (d) five.

8. Which of the following statement is correct?

   (a) Reliability ensures the validity.

   (b) Validity ensures reliability.

   (c) Reliability and validity are independent of each other.

   (d) Reliability does not depend on objectivity.

9. Authenticity of a research finding is its:

   (a) Originality.

   (b) Validity.

   (c) Objectivity.

   (d) All of the above.

10. A research problem is not feasible only when:

(a) It is researchable.

(b) It consists of independent and dependent variables.

(c) It is new and adds something to the knowledge.

(d) It has utility and relevance.

11. The research that applies the laws at the time of field study to draw more and more clear ideas about the problem is:

(a) Applied research.

(b) Action research.

(c) Experimental research.

(d) None of these.

12. Which of the following features are considered as critical in qualitative research?

(a) Gathering data with top-down schematic evidence.

(b) Collecting data with the help of standardized research tools.

(c) Design sampling with probability sample techniques.

(d) Collecting data with bottom-up empirical evidence.

13. In order to pursue the research, which of the following is on priority basis?

(a) Formulating a research question.

(b) Deciding about the data analysis procedure.

(c) Developing a research design.

(d) Formulating a research hypothesis.

14. What do you understand by the term "Anusandhan"?

(a) Goal-oriented.

(b) Attaining an aim.

(c) Praying to achieve an aim.

(d) Following an aim.

15. For the function $f(z) = (z - sinz)/z^3$, $z = 0$ is ____

(a) a pole of order 3.

(b) an isolated singularity.

(c) an essential singularity.

(d) a removable singularity

16. If Lagrangian function for a dynamical system is $L = a\dot{x}^2 + b\dot{y}^2 - kxy$ then the Hamiltonian function will be

(a) $H = \dfrac{p_x^2}{4a} + \dfrac{p_y^2}{4b} + kxy.$

(b) $H = \dfrac{p_x^2}{2a} + \dfrac{p_y^2}{2b} + kxy.$

(c) $H = \dfrac{p_x^2}{2a} + \dfrac{p_y^2}{2b} - kxy.$

(d) $H = \dfrac{p_x^2}{4a} + \dfrac{p_y^2}{4b} - kxy.$

17. Singular solution of $y = px + \sqrt{1 + p^2}$ is (where $p = \frac{dy}{dx}$) is

(a) $x^2 + y^2 = -1.$

(b) $x^2 + y^2 = 1.$

(c) $x^2 - y^2 = -1.$

(d) $x^2 - y^2 = 1.$

18. Which of the following is not an example of a path-connected space?

(a) Topologist's sine curve.

(b) Real line.

(c) Unit sphere.

(d) Euclidean space.

19. Let $Y = [0, 1) \cup \{2\}$ be a metric subspace of $\mathbb{R}$. Then,

(a) $[0, 1)$ is open but not closed in Y.

(b) $[0, 1)$ is closed but not open in Y.

(c) $[0, 1)$ is both open and closed in Y.

(d) $[0, 1)$ is neither open nor closed in Y.

Now, choose the correct option:
(i) All the statements are correct.
(ii) Only (a) is correct.
(iii) Only (b) is correct.
(iv) Only (c) is correct.

20. Let M, N be real matrices such that MN makes sense and M^T denotes the transpose of M. Then

(a) $(MN)^T = N^T M^T$.

(b) $(MN)^T = M^T N^T$.

(c) $(MN)^T = -M^T N^T$.

(d) None of the above.

21. Let $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ be a real 3×3 matrix. Choose the correct options.

(a) The rank of A is equal to the rank of $2A^T$, where A^T is the transpose of A.

(b) A may not have any characteristic root in $\mathbb{R}$.

(c) If A is nilpotent then $A+I$ is also nilpotent, where I is the identity 3×3 real matrix.

(d) A is similar to the $-A$.

22. The series $\sum_{1}^{\infty} \frac{(-1)^n \sin nx}{n^{log_e n}}, x \in \mathbb{R}$ converges

(a) Only for $x = 0$.

(b) Uniformly only for $x \in [-\pi, \pi]$.

(c) Uniformly only for $x \in \mathbb{R} \backslash \{n\pi : n \in \mathbb{Z}\}$.

(d) Uniformly for all $x \in \mathbb{R}$.

23. Let (X, d) be a usual metric space and X is the open interval $(0, 1)$. Then for a sequence defined in X by $(x_n) = 1/n$ is:

(a) a Cauchy sequence in X.

(b) it will converge in X.

(c) X is a complete metric space.

(d) X is a compact metric space.

24. The first four terms of the recurrence relation $a_k = 2a_{k-1} + k$ for all integers $k \geq 2, a_1 = 1$ are:

(a) 1, 4, 11, 26.

(b) 1, 3, 9, 21.

(c) 1, 5, 15, 25.

(d) 1, 4, 13, 23.

25. Given the matrix $A = \begin{bmatrix} 8 & -4 & 2 \\ 4 & 0 & 2 \\ 0 & -2 & -3 \end{bmatrix}$ has an eigen value of 4 with the corresponding eigen vector of $X = \begin{bmatrix} -4.5 \\ -4 \\ 1 \end{bmatrix}$ then A^5X is ____

(a) $\begin{bmatrix} -18 \\ -16 \\ 4 \end{bmatrix}$.

(b) $\begin{bmatrix} -4.5 \\ -4 \\ 1 \end{bmatrix}$.

(c) $\begin{bmatrix} -4608 \\ -4096 \\ 1024 \end{bmatrix}$.

(d) $\begin{bmatrix} -0.004395 \\ -0.003906 \\ 0.0009766 \end{bmatrix}$.

26. Suppose a second-order linear partial differential equation (PDE) is hyperbolic in a domain D. Which of the following statements is true?

(a) It is also hyperbolic in another domain D_1.

(b) It may be parabolic in another domain D_1.

(c) The PDE is hyperbolic in D_1 if D_1 is contained within D.

(d) None of the above.

27. If m and n are two positive integers such that m divides n and U_m, U_n are the corresponding groups of roots of unity, then which of the following is true?

(a) U_m is not a subgroup of U_n.

(b) U_m is a subgroup of U_n.

(c) U_n is a subgroup of U_m.

(d) U_m and U_n are disjoint groups.

28. Which of the following are correct?

(a) It is possible to construct a field of order 651.

(b) It is impossible to construct a field of order 651.

(c) There is only one ring upto isomorphism of order 651.

(d) There are more than two rings upto isomorphism of order 651.

Group B

1. Prove that $u(x, y) = e^{-x}[(x^2 - y^2)\cos y + 2xy \sin y]$ is harmonic and determine the analytic function, whose real part is u. [10]

2. Derive Hamilton's equations of motion for a compound pendulum. [10]

3. Show that if a topological space X is Hausdorff, then every finite subset of X is closed. Is the converse true? Justify your answer. [10]

4. Given a function $f: \mathbb{R} \to \mathbb{R}$, define two sets A and B in $\mathbb{R}^2$ as follows:

$$A = \{(x, y) | y < f(x)\}, B = \{(x, y) | y > f(x)\}$$

Prove that f is continuous $\iff$ both A and B are open subsets of $\mathbb{R}^2$. [10]

5. (a) Find the matrix for $\frac{d}{dx}$ acting on the vector space V of real polynomials of degree 2 or less in the ordered basis $B = \{x^2 + x, x^2 - x, 1\}$. [4]

   (b) Let

$$C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

   be a 2×2 real matrix. Prove that 2×2 matrices A and B can be found such that $C = AB - BA$ if and only if $C_{11} + C_{22} = 0$. [4]

   (c) Let $S = \{v_1, v_2, \ldots v_n\}$ be a subset of a vector space V. Show that if every vector w in V can be expressed uniquely as a linear combination of vectors in S, then S is a basis of V. [2]

6. (a) Find the conditions under which an element $v = (v_1, v_2, v_3) \in \mathbb{R}^3$ lies in the subspace generated by $(2, -3, 1)$ and $(0, 2, -1)$ in $\mathbb{R}^3$. [5]

   (b) Let V be a finite dimensional vector space over a field $\mathbb{F}$ of dimension n and let T be a linear operator on V. If T has n distinct eigenvalues (characteristic roots) in $\mathbb{F}$, then show that T is diagonalizable. [5]

7. (a) A bounded sequence $\langle f_n \rangle$ converges to l if and only if $lim_{n\to\infty} sup f_n = lim_{n\to\infty} inf f_n = l$. [5]

   (b) Evaluate the value of θ, that appears in Lagrange's mean value theorem for the function x^2–$2x + 3$ given that $a = 1$ and $b = 1/2$. [5]

8. (a) Let B and B' be Banach spaces. Let T be a linear transformation of B into B', then show that T is continuous mapping if and only if it's graph is closed. [5]

(b) Let N be a normed linear space and F denote $\mathbb{C}$ or $\mathbb{R}$. Then show that the mappings $f : N \times N \to N : f(x, y) = x + y$ and $g : F \times N \to N : g(\alpha, x) = \alpha x$ are continuous. [5]

9. Let $(L, \leq)$ be a lattice and $a, b, c \in L$. Then show that $a \vee (a \wedge b) = a$ and $a \wedge (a \vee b) = a$. [10]

10. Find the solution of the following ODE using Cauchy's Method $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log x$. [10]

11. Consider the problem

$$\begin{aligned} u_t - u_{xx} - hu = 0, 0 < x < \pi, t > 0, \\ u(0, t) = u(\pi, t) = 0, t \geq 0, \\ u(x, 0) = u(\pi - x), 0 \leq x \leq \pi, \end{aligned}$$

where h is a real constant. Solve the problem using the method of eigen function expansion. [10]

12. The following table gives the population data of India from the year 1981 to 2011. Apply Newton divided difference interpolation to find the approximate population in 2006.

Year (x)	1981	1991	2001	2011
Population (y) (in Crores)	68.3329	84.6421	102.8737	121.0193

[10]

13. (a) Show that any integral domain has a subring which is isomorphic to $\mathbb{Z}$ or $\mathbb{Z}_p$, for some prime p. [5]

(b) Let G be a group such that $(ab)^p = a^p b^p$ for all $a, b \in G$, where p is a prime number. Let $S = \{x \in G \mid x^{p^m} = e \text{ for some } m \text{ depending on } x\}$. Prove that S is a normal subgroup of G. Also show that if $\bar{G} = G/S$ and if $\bar{x}^p = \bar{e}$, then $\bar{x} = \bar{e}$. [5]

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Form No	Name	Gender	Category
230002796	SHANIAHLANG KYMPAD	MALE	ST
230002864	WANDAHUN KHARUMNUID	FEMALE	ST
230002881	PUSHAM RITA SAHANI	FEMALE	OBC (CENTRAL LIST, NON CREAMY)
230003009	RAPBORLANG SYIEM	MALE	ST
230003019	MAPHISHA NONGSIEJ	FEMALE	ST
230003036	KWI LANGSTANG	FEMALE	ST
230003101	SAIMA DEBBARMA	FEMALE	ST
230003131	SHEMPHANG NONGSHLI	MALE	ST
230003148	SAGAR PAL	MALE	OBC (CENTRAL LIST, NON CREAMY)
230003210	PREETI JOSHI	FEMALE	GENERAL
230003265	WALLAMJINGSHAI KHARDEWSAW	MALE	ST
230003345	ANUBHAB BHANDARI	MALE	GENERAL
230003419	BIJOY KALWAR	MALE	GENERAL
230003420	NEHA PRADHAN	FEMALE	OBC (CENTRAL LIST, NON CREAMY)
230003565	ABHIJIT SHARMA	MALE	OBC (CENTRAL LIST, NON CREAMY)
230003569	HAPPINESS DKHAR	FEMALE	ST
230003581	SAGOLSEM SARIDA	FEMALE	OBC (CENTRAL LIST, NON CREAMY)
230003625	BIKIRAN DAS	MALE	GENERAL
230003662	SAMUEL DKHAR	MALE	ST
230003678	BISWAJIT BARUAH	MALE	EWS
230003820	DHARMENDRA KUMAR	MALE	SC
230003876	NOURIKA SHOBNAM	FEMALE	GENERAL
230003879	BANJOPLANG SHANGDIAR	MALE	ST
230003898	KHRAW KUPAR PYNGROPE	MALE	ST
230003921	ROKOVINO CHACHEI	FEMALE	ST
230003928	DEIBORLANGKI DKHAR	MALE	ST
230003968	KABITA SHARMA	FEMALE	EWS
230004027	ASHWINI RAI	FEMALE	OBC (CENTRAL LIST, NON CREAMY)
230004032	CHEBAT A MARAK	MALE	ST
230004120	PRANJAL	MALE	SC
230004226	COLIOUS LYNGDOH	MALE	ST
230004237	CHANDAN SHARMA	MALE	GENERAL
230004259	SUBHAJIT PAUL	MALE	GENERAL
230004309	SOFIKUL ISLAM	MALE	EWS
230004311	REHENA MALICK	FEMALE	EWS
230004351	ARUNDHATI MANDAL	FEMALE	SC
230004356	KRISHNAMONI MEDHI	FEMALE	EWS
230004362	PARTHA PRATIM DAS	MALE	EWS
230004388	ANWESHA PAUL CHOUDHURY	FEMALE	GENERAL
230004435	KISHOR KUMAR	MALE	OBC (CENTRAL LIST, NON CREAMY)
230004473	BAROMETER NONGBRI	MALE	ST
230004596	HONESTAR NONGDHAR	MALE	ST
230004675	VILIHO CHISHI	FEMALE	ST
230004784	BADAPKMENHUN P DIENGDOH	FEMALE	ST
230005069	PRANJIT BORAH	MALE	OBC (CENTRAL LIST, NON CREAMY)
230005322	TEMJENSANGBA	MALE	ST
230005326	PALLAB KUMAR BORAH	MALE	GENERAL
230005417	ANGANA CHOUDHURY	FEMALE	OBC (CENTRAL LIST, NON CREAMY)
230005494	DEBDUT SENGUPTA	MALE	GENERAL

230005658	BANSIEWDORSHISHA LYNGKHOI	FEMALE	ST
230005668	SEHUTO KEZO	MALE	ST
230005758	SWRJIMA HAINARY	FEMALE	ST
230005761	ABEL BAREH	MALE	ST
230006019	NAZIMA NASRIN	FEMALE	GENERAL
230006066	SUKLA SARADI MAHANTA	FEMALE	GENERAL
230006116	SIMI BEZBARUAH	FEMALE	GENERAL
230006299	DIGANTA BORDOLOI	MALE	GENERAL
230006305	KAMAL BASUMATARY	MALE	ST
230006683	TUSHAR HALDER	MALE	OBC (CENTRAL LIST, NON CREAMY)
230006761	RAMDAY SUTRADHAR	MALE	SC
230006775	MANISHA KATHAR	FEMALE	ST
230006824	NONGMAITHEM MANOJ MEITEI	MALE	OBC (CENTRAL LIST, NON CREAMY)
230006898	LOBSANG TASHI	MALE	ST