पूर्वोत्तर पर्वतीय विश्वविद्यालय
जिलाः पूर्वी खासी हिल्स, शिलांग - 793022
मेघालय (भारत)

North-Eastern Hill University
District: East Khasi Hills, Shillong - 793022
Meghalaya (India)

No. ACAD/81/1911

Date: 19-10-2023.

NOTICE

There will be an Entrance test for admission into the Ph.D. programme in Mathematics on the 15th November, 2023 from 10.30 a.m. to 1.30 p.m. in Mathematics Department, NEHU Shillong. The list of eligible candidates for the entrance test and a model question paper are appended below.

Only those who score minimum 50% marks in the written test (5% concession for SC/ST/OBC (Non-creamy layer)) will be called for an interview to be held on 16th November, 2023 in the Conference Room of the Department. The time of the interview will be notified later.

19-10-2023.

Head, Department of Mathematics
NEHU, Shillong.

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LIST OF ELIGIBLE CANDIDATES TO APPEAR THE ENTRANCE TEST TO PH.D. IN MATHEMATICS, NEHU SHILLONG FOR THE YEAR 2023

Sl No.	Application No	Form Serial No.	Name
1	23490018	132533	Delizia Jones Tariang
2	23490062	133235	Jyoti Prasad Ray
3	23490112	133744	Shemphang Nongshli
4	23490160	134406	omen jamatia
5	23490321	135887	Steward Pastieh Pakma
6	23490412	136751	Gordon Oniel Kharpuri
7	23490469	137173	Mebanker Nanglein
8	23490521	137508	Liza HAZARIKA
9	23490691	138429	Diwanson Shylla
10	23490750	138703	Betsheba Ch Momin
11	23490889	139443	Daphimosha Lyngdoh
12	23490919	139606	David Allison Lyngdoh
13	23490953	139774	Nilutpal Saikia
14	23491004	140087	Jeremy Rymbai

15	23491014	140168	Mridu Paban pathak
16	23491070	140965	Palmei Chingkhu
17	23491173	141987	Ritwik Prabin Kalita
18	23491438	143254	Rupashree Das
19	23491447	143278	Nisorsingh Kharlartang
20	23491484	143402	Manas Pratim Kalita
21	23491511	143465	BIJETA PAUL
22	23491542	143533	ABHISHEK GIRISH AHER
23	23491595	143689	Habanjop Kurbah
24	23491626	143757	ANUBHAB BHANDARI
25	23491628	143759	MAMTA TUDU
26	23491772	144104	TAMDING WANGCHUK
27	23491813	144200	SUBHADIP BHOWMIK
28	23491849	144297	C. Tengrak R Marak
29	23491864	144337	Didara Khongpdah
30	23491896	144394	Diganta Bordoloi
31	23491910	144435	BONYFIRST SIANGSHAI

32	23491934	144474	Nungsangti M Lemtor
33	23491948	144500	Krishna Changmai
34	23491978	144555	Shamiran Sarmah Bordoloi
35	23492016	144640	SUJATA SAIKIA
36	23492044	144690	NSIMTUBE NDANG
37	23492051	144706	ANINDYA KUNDU
38	23492104	144798	Nangsankupar Lyngdoh Mawlong
39	23492187	145001	Rousel Wilson Shabong
40	23492212	145029	Masuma Akter
41	23492296-I	145122	BASTEN Sharwell GIROD
42	23492309	145137	WINNE BAREH
43	23492325	145159	SWRJIMA HAINARY
44	23492348	145190	AKASHEE BORAH
45	23492477	145333	Samriddha Deb
46	23492494	145353	DIPANJALI BASUMATARY
47	23492558	145423	ALMINA AHMED
48	23492615	145484	Rajarshi Saraswati

Registration Number

NORTH EASTERN HILL UNIVERSITY

SCHOOL OF PHYSICAL SCIENCES

DEPARTMENT OF MATHEMATICS
PhD ENTRANCE EXAMINATION

Model Test Paper

ACADEMIC YEAR: 2023/2024
TIME: 3 HRS
MAXIMUM MARKS: 100

INSTRUCTIONS:

1. This paper consists of **SIX (5)** printed pages.
2. There are **TEN (10)** questions in **Section (A)**. All questions are compulsory in this section.
3. In **Section (B)**, there are **TEN (10)** questions. Attempt any **FIVE (5)** from this section.
4. There are **TEN (10)** questions in **Section (C)**. Attempt any **SIX (6)** in this section.
5. Do not use this test paper for rough work. All rough work must be done in the answer book (at the back) and crossed through.
6. This test paper must be handed in together with your answer book.
7. Unauthorized materials and gadgets such as mobile phones and pagers, programmable pocket calculators are **NOT** allowed in the examination venues.

Section-A

Note: All questions in this section are compulsory. **(2 x 10=20 marks)**

1. Let R be a commutative ring and I be an ideal of R such that $I \neq R$. Then which of the following is correct?
   (a) If R is an integral domain, then the quotient ring R/I is an integral domain.
   (b) If R/I is an integral domain, then R is an integral domain.
   (c) If R is an integral domain, then I is a prime ideal of R.
   (d) If R is a PID, then R/I is a PID.

2. Let A be an open subset of $\mathbb{R} \times \mathbb{R}$. Two points x, $y \in A$ are said to be equivalent if x can be joined to y by a continuous path completely lying inside A. Then which of the following is true:
   The number of equivalence classes is
   (a) Only one.
   (b) At most finite.
   (c) At most countable.
   (d) Can be finite, countable or uncountable.

3. A real valued function f is continuous in the closed interval $0 \leq x \leq 1$ and differentiable in open interval $0 < x < 1$. Then which of the following is true for some $c, 0 < c < 1$.:
   (a) $f(1) = f(0) = f'(c)$
   (b) $f'(c) = f(1) - f(0)$
   (c) $f(c) = f(1) - f(0)$
   (d) none of these.

4. The series whose n^{th} term is given by $a_n = \frac{1}{logn}$ is
   (a) convergent
   (b) divergent
   (c) neither convergent nor divergent
   (d) none of these.

5. The only subspaces of the vector space $\mathbb{R}$ over $\mathbb{R}$ are:
   (a) $\mathbb{R}$ and the zero subspace.
   (b) $\mathbb{Q}, \mathbb{R}$ and the zero subspace.
   (c) $\mathbb{Z}, \mathbb{Q}, \mathbb{R}$ and the zero subspace.
   (d) None of the above.

6. What is the order of convergence of Regula-Falsi method ?
   (a) 2.231
   (b) 2.312

(c) 1.618

(d) 1.321.

7. If the functional $f(x, y, y^{'})$ does not contain x explicitly, then the Euler equation is given by

(a) $f_y - \frac{d}{dx} f_{y'} = 0$

(b) $f - y^{'} f_{y'} = constant$

(c) $f_y - y^{'} f_{y'} = constant$

(d) $f_y - y f_{y'} = constant$

8. The image of the circle $|z-3i| = 3$ in the complex plane under the mapping $w = \frac{1}{z}$, (where, $w = u + iv$) is

(a) $6u - 1 = 0$

(b) $6v + 1 = 0$

(c) $6u + 1 = 0$

(d) $6v - 1 = 0$.

9. The solution of initial value problem $y''' - 4y' = 0, y(0) = 0, y'(0) = 1, y''(0) = 1$ is:

(a) $\phi(x) = \frac{sinh2x}{2}$

(b) $\phi(x) = \frac{cosh2x}{2}$

(c) $\phi(x) = \frac{sin2x}{2}$

(d) $\phi(x) = \frac{cos2x}{2}$

10. The particlular integral of $\frac{1}{(D+2D')} e^{2x+y}$ where $D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$ is equal to

(a) 2^{2x+y}

(b) $\frac{1}{3} e^{2x+y}$

(c) $\frac{1}{2} x^2 e^{2x+y}$

(d) $x e^{2x+y}$

Section-B

Note: Attempt any five (5) questions from this section. Answer should be supported by a brief justification. (4x5 =20 marks)

1. Is it true that there exists a non-abelian group all of whose subgroups are normal?

2. Let $f \colon X \to Y$ be a continuous map. (Here X, Y are topological spaces with Y Hausdorff). Show that the set $\{(x, y) | f(x) = f(y)\}$ is a closed subset of $X \times X$.

3. Prove that

$$lim_{n \to \infty} [(1 + 1/n)(1 + 2/n)....(1 + 4n/n)]^{1/n} = 5^5/e^4.$$

4. Let X be the set of all real valued bounded continuous functions defined on the closed interval $[0, 1]$. We define the norm of $f \in X$ by

$$\parallel f \parallel = \int_0^1 |f(x)| dx.$$

Let d be a mapping of $X \times X$ into $\mathbb{R}$ defined by $d(f, g) = \parallel f - g \parallel = \int_0^1 |f(x) - g(x)| dx, \forall f, g \in X$. Then show that d is a metric on X.

5. Let N be a normed linear space and let $x, y \in N$. Then show that

$$| \parallel x \parallel - \parallel y \parallel | \leq \parallel x - y \parallel .$$

6. For a fixed $c \in \mathbb{R}$, let $\alpha = \int_0^2 (9x^2 - 5cx^4) dx$. If the value of $\alpha = \int_0^2 (9x^2 - 5cx^4) dx$ is obtained by using the Trapezoidal rule is also equal to α. Then find the value of α.

7. Show that the Poissons bracket is canonical invariant.

8. Evaluate $\int_C \frac{(12z-7)dz}{(2z+3)(z-1)^2}$, where C is $x^2 + y^2 = 4$.

9. Check whether x and xe^x are linearly independent or not.

10. Solve: $py + qx = xyz^2(x^2 - y^2)$.

Section-C

Note: Attempt any six (6) questions from this section. Answer should be supported by a full justification. (6x10=60 marks)

1. If G is a group of order 10 and $H = \{x \in G : o(x) \text{ is odd}\}$, prove that H is a normal subgroup of G.

2. Show that every compact Hausdorff space is normal. Is every normal space is a compact Hausdorff space?

3. Let $H = \{x \in \mathbb{R}^n : a.x \geq t\}, a \neq 0 \in \mathbb{R}^n$. Then show that H is a convex subset of $\mathbb{R}^n$.

4. Let B and B' be Banach spaces. If T is a continuous linear transformation of B onto B', then show that T is an open mapping.

5. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear operator whose matrix representation with respect to the standard basis is given by

$$\begin{bmatrix} 0 & 0 & 0 \\ a & 0 & 0 \\ 0 & b & 0 \end{bmatrix}$$

For what values of a and b is T diagonalizable?

6. Let A be the following invertible matrix with real positive entries:

$$A = \begin{bmatrix} 1 & 2 \\ 8 & 9 \end{bmatrix}.$$

Let G be the associated Gauss-Seidel iteration matrix. What are the eigen values of A ?

7. Using Lagrange's equation to find the differential equation for a compound pendulum which oscillates in a vertical plane about a fixed horizontal axis.

8. Prove that $u(x, y) = e^{-x}[(x^2 - y^2)cosy + 2xysiny]$ is harmonic and determine the analytic function whose real part is u.

9. If one of the solution of the ordinary differential equation $x^2y'' - 7xy' + 15y = 0$ for $x > 0$ is $\phi_1(x) = x^3$. Then find the other independent solution.

10. State and discuss the solution procedure of linear partial differential equation of order one using Lagrange's method.
