

**Detailed Proposal for
AIS in Differential Topology 2018**

Name of School : AIS in Differential Topology

Venue : Department of Mathematics, North-Eastern Hill University, NEHU Campus, Shillong-793022.

Dates : July, 02 – July, 21, 2018.

Names of Organizers :

1. Himadri Kumar Mukerjee,
Department of Mathematics,
North-Eastern Hill University,
NEHU Campus, Shillong-793022, Meghalaya, India.
2. Angom Tiken Singh
Department of Mathematics,
North-Eastern Hill University,
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Names of Possible resource persons:

Amiya Mukherjee, ISI, Kolkata
Anant R. Shastri, IIT Bombay, Mumbai
Angom Tiken Singh, NEHU, Shillong
B. Subhash, IISER Tirupati, Tirupati
Hemant Kumar Singh, Delhi University, Delhi
Himadri Kumar Mukerjee, NEHU, Shillong

Names of Possible Standby Resource persons/course associates:

S. Maitra, RD University, Jabalpur
Shilpa Suresh Gondhali, BITS, Goa
Jiban Singh, Assam University, Silchar
Swagata Sarkar, CEBS, Mumbai

Target Audience : Ph. D. students and young faculty in universities, research institutes etc in the country, who would like to acquire a basic working knowledge of Differential Manifolds.

A brief Description of the school for publicity (200 words)

Introduction: [Hermann Weyl](#) gave an intrinsic definition for differentiable manifolds in 1912. Differential and integral calculus carry over to differential manifolds in a suitable way. During the 1930s [Hassler Whitney](#) and others clarified the [foundational](#) aspects of the subject, and thus intuitions dating back to the latter half of the 19th century became precise, the [Whitney embedding theorem](#), that manifolds intrinsically defined by charts could always be embedded in Euclidean space, as in the extrinsic definition, showed that the two concepts of manifold were equivalent. Differential Topology mainly deals with the global properties of differentiable manifolds. In dealing with problems of classification and intersection theory of such manifolds one often has to bring in tools and techniques from algebraic topology.

The purpose of the AIS would be to expose some of these topics in a condensed and yet lucid fashion and help students in digesting the topics through tutorials; this will prepare them to take up research work on the subject.

Tentative syllabus:

1. Topological manifolds, differentiable structures, smooth manifolds, smooth functions and smooth maps, tangent vectors and tangent space at a point, derivatives of a smooth map, inverse, implicit and rank theorems, submanifolds, regular and critical points and values, Sard's theorem (special cases), immersion and submersion, inverse image of regular values. (First week)
2. Whitney's embedding theorem, transversality, inverse image of transverse submanifolds, transversality theorem, vector bundles, tangent and normal bundles, Grassmann manifolds, universal bundles, universality of the universal bundle. (Second week)
3. Thom's cobordism theory, definition of cobordism groups, Thom spaces, Thom's isomorphism theorem. (Third week)
4. Review of singular homology of spaces and pairs, Mayer-Vietoris sequence, adjunction spaces, cell complexes, singular cohomology, calculation of (co)homology groups of cell complexes and standard manifolds, Cup and cap products. (First week)
5. Local (co)homology of manifolds, orientation covers, orientation of manifolds, fundamental class of a compact orientable manifold, the Poincare, Alexander and Lefschetz duality theorems (statements, explanations and applications only). (Second week)
6. Geometric and algebraic intersection theory, the Lefschetz fixed point theorem, the Hopf index theorem, framed manifolds, framed cobordism, Hopf degree theorem. (Third week)

Syllabus (Some addition/subtraction may be made in the topics on the advice of resource persons):

	Lectures	Detailed Syllabus
Topic 1 (HKS)	6 (90 minute)	Topological manifolds, differentiable structures, smooth manifolds, smooth functions and smooth maps, tangent vectors and tangent space at a point, derivatives of a smooth map, inverse, implicit and rank theorems, submanifolds, regular and critical points and values, Sard's theorem (special cases), immersion and submersion, inverse image of regular values. (First week)
Topic 2 (ATS)	6 (90 minute)	Whitney's embedding theorem, transversality, inverse image of transverse submanifolds, transversality theorem, vector bundles, tangent and normal bundles, Grassmann manifolds, universal bundles, universality of the universal bundle.
Topic 3 (HKM)	6 (90 minute)	Thom's cobordism theory, definition of cobordism groups, Thom spaces, Thom's isomorphism theorem.
Topic 4 (BS)	6 (90 minute)	Review of singular homology of spaces and pairs, Mayer-Vietoris sequence, adjunction spaces, cell complexes, singular cohomology, calculation of (co)homology groups of cell complexes and standard manifolds, Cup and cap products.
Topic 5 (AM)	6 (60 minute)	Local (co)homology of manifolds, orientation covers, orientation of manifolds, fundamental class of a compact orientable manifold, the Poincare, Alexander and Lefschetz duality theorems (statements, explanations and applications only).
Topic 6 (ARS)	6 (60 minute)	Geometric and algebraic intersection theory, the Lefschetz fixed point theorem, the Hopf index theorem, framed manifolds, framed cobordism, Hopf degree theorem.

AM – Amiya Mukherjee (ISICAL)
 ARS – Ananta R. Shastri (IITB)
 ATS - Angom Tiken Singh (NEHU)
 BS – B. Subhash (IISER Tirupati)
 HKM – Himadri Kumar Mukerjee (NEHU)
 HKS - Hemant Kumar Singh (DU)

Time-Table:

		Lecture 1	Tea	Lecture 2	Lunch	Tutorials	Tea	Tutorials	Tea and Snacks
Day	Date	9.30 -11.00	11 - 13	11.30 -13.00	13 -15	15.00 - 16.00	16 -16.30	16.30 – 17.30	17.30
Monday	02-07-2018	Topic 1 (HKS)		Topic 4 (BS)		Tutorial (Topic 1)		Tutorial (Topic 4)	
Tuesday	03-07-2018	Topic 1 (HKS)		Topic 4 (BS)		Tutorial (Topic 1)		Tutorial (Topic 4)	
Wednesday	04-07-2018	Topic 1 (HKS)		Topic 4 (BS)		Tutorial (Topic 1)		Tutorial (Topic 4)	
Thursday	05-07-2018	Topic 1 (HKS)		Topic 4 (BS)		Tutorial (Topic 1)		Tutorial (Topic 4)	
Friday	06-07-2018	Topic 1 (HKS)		Topic 4 (BS)		Tutorial (Topic 1)		Tutorial (Topic 4)	
Saturday	07-07-2018	Topic 1 (HKS)		Topic 4 (BS)		Tutorial (Topic 1)		Tutorial (Topic 4)	
Sunday	08-07-2018	Break		Break		Break			
Monday	09-07-2018	Topic 2 (ATS)		Topic 5 (AM)		Tutorial (Topic 2)		Tutorial (Topic 5)	
Tuesday	10-07-2018	Topic 2 (ATS)		Topic 5 (AM)		Tutorial (Topic 2)		Tutorial (Topic 5)	
Wednesday	11-07-2018	Topic 2 (ATS)		Topic 5 (AM)		Tutorial (Topic 2)		Tutorial (Topic 5)	
Thursday	12-07-2018	Topic 2 (ATS)		Topic 5 (AM)		Tutorial (Topic 2)		Tutorial (Topic 5)	
Friday	13-07-2018	Topic 2 (ATS)		Topic 5 (AM)		Tutorial (Topic 2)		Tutorial (Topic 5)	
Saturday	14-07-2018	Topic 2 (ATS)		Topic 5 (AM)		Tutorial (Topic 2)		Tutorial (Topic 5)	
Sunday	15-07-2018	Break		Break		Break			
Monday	16-07-2018	Topic 3 (HKM)		Topic 6 (ARS)		Tutorial (Topic 3)		Tutorial (Topic 6)	
Tuesday	17-07-2018	Topic 3 (HKM)		Topic 6 (ARS)		Tutorial (Topic 3)		Tutorial (Topic 6)	
Wednesday	18-07-2018	Topic 3 (HKM)		Topic 6 (ARS)		Tutorial (Topic 3)		Tutorial (Topic 6)	
Thursday	19-07-2018	Topic 3 (HKM)		Topic 6 (ARS)		Tutorial (Topic 3)		Tutorial (Topic 6)	
Friday	20-07-2018	Topic 3 (HKM)		Topic 6 (ARS)		Tutorial (Topic 3)		Tutorial (Topic 6)	
Saturday	21-07-2018	Topic 3 (HKM)		Topic 6 (ARS)		Tutorial (Topic 3)		Tutorial (Topic 6)	

References:

1. BREDON G., Topology and Geometry, Springer 1993. This book introduces many of the tools needed for manifold surgery.
2. GAULD David B., Differential Topology: An Introduction, Dover Publications, 2006.
3. GREENBERG and HARPER, Algebraic Topology – a first course, 1981.
4. GUILLEMIN Victor and POLLACK Alan, Differential Topology, AMS, 2010.
5. HIRSCH M.W., Differential Topology, Springer-Verlag, 1997.
6. HUSEMULLER Dale, Fibre bundles, GTM-20, Springer, 1994.
7. KOSINSKI, A.A., Differential manifolds, Academic Press, 1993 and Dover publications, 2007.
8. MILNOR J., "Differential Topology", Volume 3 of his collected papers, Amer. Math. Soc. 2007.
9. MILNOR J., Topology from the Differentiable Viewpoint, Univ. Pross of Virginia at Charlottesville, 1965.
10. Mukherjee, Amiya, Topics in Differential Topology, Hindistan Book Agency, 2005.
11. Shastri, A.R., Elements of Differential Topology, CRC Press, 2011.
12. Shastri, A.R., Basic Algebraic Topology, CRC Press, 2013.
13. STEENROD Norman J. , Topology of Fibre bundles, Princeton, 1951; Landmarks in Mathematics and Physics; Princeton, 1999.
14. WALLACE A.H., Differential Topology: First Steps, Dover Books on Mathematics, 2006.

There are many more new books and resources in the internet.

Contact Details of Organizers

(Mobile, home and office telephone numbers, Fax, Mailing addresses)

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Contact Details and address of the person in whose favour cheque is to be sent

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