

SYLLABUS
FOR
M. Sc. (MATHEMATICS)
UNDER
NEP2020

**SYLLABUS STRUCTURE OF M.SC. PROGRAMME IN
MATHEMATICS UNDER NEP 2020**

FIRST SEMESTER			
Paper Code	Paper	No. of papers	Total Credit
MTH-CC-500	Common Course(CC) Analysis-I	1	4
MTH-CC-501	Linear Algebra	1	4
MTH-DSEC-502	Discipline-Specific Elective Course (DSEC) Ordinary Differential Equations	1	4
MTH-DSEC-503	Classical Mechanics	1	4
MTH-GEC-504	Generic Elective Course (Multidisciplinary) Applications of Mathematics in Environmental studies	1	4
Total		5	20
SECOND SEMESTER			
Paper Code	Paper	No. of papers	Total Credit
MTH-CC-505	Common Course(CC) Algebra	1	4
MTH-CC-506	Analysis II	1	4
MTH-DSEC-507	Discipline-Specific Elective Course (DSEC) Topology	1	4
MTH-DSEC-508	Partial Differential Equations	1	4
MTH-RM-509	Research Methodology & Proposal Writing	1	4
MTH-SEC-510	Skill Enhancement Course (SEC) (Data Analysis/Digital Learning) Programming in C	1	4
Total		6	24
THIRD SEMESTER			
Paper Code	Paper	No. of papers	Total Credit
MTH-CC-600	Common Course(CC) Complex Function Theory	3	12
MTH-CC-601	Numerical Analysis		
MTH-CC-602	Theory of Field Extensions		
MTH-DSEC-603	Discipline-Specific Elective Course (DSEC) - Students can opt any three out of the following DSEC courses and courses offered by MOOCS/SWAYAM after verification by the department so as to avoid duplicity	3	12
MTH-DSEC-604	Graph Theory	3	12
MTH-DSEC-605	Ring Theory		
MTH-DSEC-606	Algebraic Topology		
MTH-DSEC-606	Theory of Relativity		

MTH-DSEC-607	Functional Analysis Differential Geometry Number Theory Integral Equations and Calculus of Variations Riemannian Geometry		
MTH-DSEC-608			
MTH-DSEC-609			
MTH-DSEC-610			
MTH-DSEC-611			
Total		6	24
FOURTH SEMESTER			
Paper Code	Paper	No. of papers	Total Credit
MTH-612	Research Project/Dissertation	1	20
Total		1	20

Note: 1 Credit Hour is equivalent to 15 hours of teaching for theory papers and 30 hours of practical activities for Practical Papers.

Note for the paper setter to be forwarded along with the syllabus of the paper

1. Note for the paper setter:

There are four units. Examinees are to attempt question number one and four more questions taking one from each of the four units. Two questions are to be set from each unit.

Recommendation:

1) The first question will be of 27 marks and will be compulsory. In all 12 short questions of 3 marks each shall be asked taking 3 questions from each unit. Examinees have to choose 9 questions out of 12. These questions may be of type true/false.

2) The remaining questions shall be of 12 marks each.

DETAILS OF SYLLABI

MTHC-CC-500: Analysis-I

[T = Contact hours (Theory) = 60, P = Contact hours (Practical) = Nil, CR = Credits = 4, Marks=100].

UNIT I

Finite, infinite, countable and uncountable sets, Schröder-Bernstein theorem; the ordered real field, Archimedean property, density of rational numbers, existence of n^{th} root of positive real numbers, exponential and logarithm; metric spaces, open and closed sets, limit points, interior points, compact spaces; Nested interval theorem, Bolzano Weierstrass theorem, Heine-Borel theorem.

(Contact hours – 15)

UNIT II

Connected sets, connected subsets of real numbers, Sequences, Cauchy sequences, complete metric space, completeness property of \mathbb{R} , construction of real numbers using Cauchy sequences, limit supremum and limit infimum; series, series of nonnegative terms, the number e , the root and ratio tests; summation by parts, absolute convergence, addition and multiplication of series, rearrangements.

(Contact hours – 15)

UNIT III

Limits of functions, continuous functions, uniform continuity, continuity and compactness, continuity and connectedness, intermediate value theorem; one-dimensional Brouwer fixed point theorem; discontinuities and their classifications, monotonic functions, infinite limits and limits at infinity; differentiation of real valued functions and mean value theorem, differentiation of vector-valued functions; Riemann integration: change of variable, fundamental theorem of calculus, integration of vector-valued functions.

(Contact hours – 15)

UNIT-IV

Sequences and series of functions, pointwise and uniform convergence; uniform convergence and continuity; uniform convergence and integration; uniform convergence and differentiation; nowhere differentiable functions; Statement of Stone-Weierstrass' theorem for a real and complex-valued functions on an interval.

(Contact hours – 15)

Textbooks:

1. Principles of Mathematical Analysis (3th edition) – W. Rudin, McGraw Hill Kogakusha Ltd., 2017.
2. Elementary Analysis (2nd Edition) - Kenneth A. Ross, Springer (2013).

Reference books:

1. Mathematical Analysis (5th edition) – T. Apostol, Addison-Wesley; Publishing Company, 2001.
2. Introduction to Real Analysis (4th edition) – R. G. Bartle and D. R. Sherbert, John Wiley & Sons, Inc., New York, 2021.

3. Introduction to Topology- Collin Adams and Franzosa, Pearson, Prentice Hall of India (2009).
4. Basic Real Analysis (2nd edition)– H.H. Sohrab, Birkhäuser (2014).

MTH-CC-501: LINEAR ALGEBRA

[T = Contact hours (Theory) = **60**, P = Contact hours (Practical) = **Nil**, CR = Credits = **4**, Marks=100].

UNIT-I

Vector spaces, basis and dimension, linear transformations, rank-nullity theorem, matrix representation of a linear transformation; linear functionals, isomorphism between the algebra of linear transformations and that of matrices; similarity of matrices and linear transformations. (Contact hours – 15)

UNIT-II

Eigen values and eigen vectors, characteristic polynomials, relation between characteristic polynomial and minimal polynomial; Cayley-Hamilton theorem; diagonalizability, necessary and sufficient condition for diagonalizability; projections and their relation with direct sum decomposition of vector spaces; invariant subspaces. (Contact hours – 15)

UNIT-III

Primary decomposition theorem, cyclic subspaces; companion matrices; triangulability; canonical forms of nilpotent transformations; Jordan canonical forms; inner product spaces; properties of inner products and norms; Cauchy-Schwarz inequality; orthogonality and orthogonal complements, orthonormal basis, Gram-Schmidt process. (Contact hours – 15)

UNIT-IV

Adjoint of a linear transformation; Hermitian, unitary and normal transformations and their diagonalizations; forms on inner product spaces and their matrix representations; Hermitian forms; principal axis theorem, bilinear forms; symmetric bilinear forms; quadratic forms; orthogonal diagonalization of symmetric bilinear forms, signature of symmetric bilinear form. (Contact hours – 15)

Textbook:

1. Linear Algebra, Stephen A. Friedberg, A.J. Insel and L.E. Spence, Pearson (2014).
2. Linear Algebra (2nd edition) – K. Hoffman and R. Kunze, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.

Reference books:

1. Linear Algebra (2nd edition) – Promode Kumar Saikia, Pearson (second edition), 2014.
2. Topics in Algebra (4th edition) – I. N. Herstein, Wiley Eastern Limited, New Delhi, 2003.
3. First Course in Linear Algebra – P. B. Bhattacharya, S. K. Jain and S. R. Nagpal, Wiley Eastern Ltd., New Delhi, 2000.

4. Linear Algebra – G. E. Shilov, Prentice Hall, 1998.
5. Finite Dimensional Vector Spaces – P. R. Halmos, Van Nostrand Inc., 1965.
6. Introduction to Matrices and Linear Transformations (3rd edition) – D. T. Finkbeiner, D.B. Taraporevala, Bombay, 1990.
7. Linear Algebra, A Geometric Approach – S. Kumaresan, Prentice-Hall of India Pvt. Ltd., New Delhi, 2001.

MTH-DSEC-502: ORDINARY DIFFERENTIAL EQUATIONS

[**T** = Contact hours (Theory) = **60**, **P** = Contact hours (Practical) = **Nil**, **CR** = Credits = **4**,
Marks=100].

UNIT I

Linear equations of first order; linear equations with constant coefficients; the second and nth order homogeneous equation; initial value problems for second and nth order equations; existence theorem; uniqueness theorem; linear dependence and independence of solutions; the Wronskian and linear independence; a formula for the Wronskian; the non-homogeneous equation of second order and nth order. (Contact hours – 15)

UNIT II

Existence and uniqueness of solutions to the first order equation; equations with variable separated; exact equations, the method of successive approximations; Lipschitz condition; convergence of successive approximation (existence theorem); non-local existence of solutions; uniqueness of solutions; existence and uniqueness of solutions to systems and n-th order equations. (Contact hours – 15)

UNIT III

Linear equations with variable coefficients; initial value problems for the homogeneous equations; solutions of homogeneous equations; Wronskian and linear independence; reduction of the order of a homogeneous equation; non-homogeneous equations; homogeneous equations with analytic coefficients; the Legendre equation, justification of the power series method. (Contact hours – 15)

UNIT IV

Linear equations with regular singular points; the Euler equation; second order equations with regular singular points – example and the general case, a convergence proof, the exceptional cases; the Bessel equation; regular singular points at infinity; autonomous systems, the phase plane; critical point; types of critical points; stability; stable critical point; asymptotically stable; stability for linear systems; stability by Liapunov's direct method. (Contact hours – 15)

Textbooks:

1. An Introduction to Ordinary Differential Equations – E. A. Coddington, Prentice-Hall of India Private Ltd., New Delhi, 2012.
2. Differential equations with applications and historical notes (2nd edition) – G.F. Simmons, Tata McGraw-Hill, New Delhi, 2016.

Reference books:

1. Elementary Differential Equations (3rd Edition) – W. T. Martin and E. Reissner, Addison Wesley Publishing Company, inc., 1995.
2. Theory of Ordinary Differential Equations – E. A. Coddington and N. Levinson, Tata McGraw hill Publishing co. Ltd. New Delhi, 1999.
3. Differential Equations, Dynamical Systems and an Introduction to Chaos – M.W. Hirsch, S. Smale, and R.L. Devaney, Elsevier (2004).
4. Spherical Harmonics – T. M. Mac Robert, Pergamon Press, 1967.

MTH-DSEC-503: CLASSICAL MECHANICS

[T = Contact hours (Theory) = **60**, P = Contact hours (Practical) = **Nil**, CR = Credits = **4**, Marks=100].

UNIT I

Generalized coordinates; holonomic & non-holonomic systems; D'Alembert's principle; Lagrange's equations; calculus of variations. (Contact hours – 15)

UNIT II

Hamilton's principle, Lagrange's equations from Hamilton's principle, extension of Hamilton's principle to non-conservative and non-holonomic systems, conservation theorems and symmetry properties. (Contact hours – 15)

UNIT III

Eulerian angles; Euler's theorem on the motion of a rigid body; infinitesimal rotations; rate of change of a vector; coriolis force; Euler's equations of motion; force free motion of a rigid body; heavy symmetrical top with one point fixed. (Contact hours – 15)

UNIT IV

Hamilton's equations of motion, conservation theorems and physical significance of Hamiltonian, Hamilton's equations from variational principle, principle of least action; equations of canonical transformation; integral invariants of Poincare'; Lagrange and Poisson brackets as canonical invariants, equations of motion in Poisson bracket notation; infinitesimal contact transformations; constants of motion and symmetry properties. (Contact hours – 15)

Textbook:

1. Classical Mechanics (3rd edition) – H. Goldstein, Addison Wesley Publications, Massachusetts, 2002.

Reference books:

1. Classical Mechanics – C. R. Mondal, Prentice-Hall of India, 2001.
2. Classical Mechanics (5th edition)– T. W. B. Kibble, Orient Longman, London, 2004.
3. Mechanics – L. D. Landau and E. M. Lifshitz, Pergamon Press, Oxford, 1976.
4. Lectures on Mechanics – J. E. Marsden, Cambridge University Press, 1992.

MTHC-GEC-504: APPLICATION OF MATHEMATICS IN ENVIRONMENTAL STUDIES

[T = Contact hours (Theory) = 60, P = Contact hours (Practical) = Nil, CR = Credits = 4].

UNIT I

System of linear equations, matrix form, elementary row operations, row equivalence, row reduced, row reduced echelon matrices, elementary matrices and their roles in determining invertibility of square matrices, row rank, relation between row equivalence and row space of matrices, matrix population modelling. (Contact hours – 15)

UNIT-II

Linear programming problem – introduction, graphical solution method, some exceptional cases; general linear programming problem, duality, simplex method; problems related to ecology and environment. (Contact hours – 15)

UNIT III

Simple situations requiring mathematical modelling, techniques of mathematical modeling, Classifications, Characteristics and limitations of mathematical models, Some simple illustrations. Mathematical modelling in population dynamics, Mathematical modelling of epidemics through systems of ordinary differential equations of first order Mathematical Models in Medicine. (Contact hours – 15)

UNIT-IV

Discrete dynamical systems, orbit of a point, types of orbit; fixed point: sink, source and neutral fixed points; classification of fixed points of real-valued function of one real variable, dynamics of logistic population model.

(Contact hours – 15)

Textbooks:

1. Linear Algebra– K. Hoffman and R. Kunze, Prentice Hall of India Pvt. Ltd., New Delhi, (2nd edition) 2000.
2. Introduction to Applied Mathematics for Environmental Science – David F. Parkhurst, Springer (2006).
3. Operations Research (for Group B) – K. Swarup, P. K. Gupt and Man Mohan, Sultan Chand & Sons, New Delhi, 2000.
4. Mathematical Modelling- J. N. Kapur, New Age International, 1988.
5. Differential equations, dynamical systems and introduction to chaos - M. Hirsch, S. Smale & R.L. Devaney; Academic Press, 2013, Elsevier.

References:

1. Linear Algebra and its application - Gilbert Strang, Brooks Cole, 4th edition (2006).
2. Rutherford, A. *Mathematical Modelling Techniques*. Courier Corporation, 2012.
3. Linear Algebra (2nd edition) – Promode Kumar Saikia, Pearson, 2009.

MTH-CC-505: ALGEBRA

[T = Contact hours (Theory) = **60**, P = Contact hours (Practical) = **Nil**, CR = Credits = **4**, Marks=100].

UNIT I

A brief review of groups, their elementary properties and examples, subgroups, cyclic groups, homomorphism of groups and Lagrange's theorem; permutation groups, permutations as products of cycles, even and odd permutations, normal subgroups, quotient groups; isomorphism theorems, correspondence theorem. (Contact hours – 15)

UNIT II

Group action; Cayley's theorem, group of symmetries, dihedral groups and their elementary properties; orbit decomposition; counting formula; class equation, consequences for p-groups; Sylow's theorems (proofs using group actions), applications of Sylow's theorems, conjugacy classes in S_n and A_n , simplicity of A_n .

(Contact hours –15)

UNIT III

Direct product; structure theorem for finite abelian groups; invariants of a finite abelian group; basic properties and examples of ring, domain, division ring and field; direct products of rings; characteristic of a domain; field of fractions of an integral domain; ring homomorphisms (always unitary); ideals; factor rings; prime and maximal ideals, principal ideal domain.

(Contact hours – 15)

UNIT IV

Euclidean domain; unique factorization domain; a brief review of polynomial rings over a field; reducible and irreducible polynomials, Gauss' theorem for reducibility of $f(x) \in \mathbf{Z}[x]$; Eisenstein's criterion for irreducibility of $f(x) \in \mathbf{Z}[x]$ over \mathbf{Q} , roots of polynomials; finite fields of orders 4, 8, 9 and 27 using irreducible polynomials over \mathbf{Z}_2 and \mathbf{Z}_3 .

(Contact hours – 15)

Textbooks:

1. A First Course in Abstract Algebra (7th edition) – J. B. Fraleigh, Pearson Education Ltd., 2013.

Reference books:

1. Basic Abstract Algebra (South Asia edition) – P.B. Bhattacharya, S. K. Jain and S. R. Nagpal, Cambridge University Press, 2003.
2. Basic Algebra I (2nd edition) – N. Jacobson, Dover Publications, 2009.
3. Contemporary Abstract Algebra (4th edition) – J. A. Gallian, Narosa Publishing House, New Delhi, 1999.
4. An Introduction To The Theory of Groups (4th edition) – J. J. Rotman, Allyn and Bacon, Inc., Boston, 2002.
5. Topics in Algebra (4th edition) – I. N. Herstein, Wiley Eastern Limited, 2003.
6. Abstract Algebra – D.S. Dummit, R.M. Foote, John Wiley&Sons (2003).

MTH-CC-506: ANALYSIS-II

[T = Contact hours (Theory) = **60**, P = Contact hours (Practical) = **Nil**, CR = Credits = **4**, Marks=100].

UNIT-I

σ -rings of sets, Borel sets, additive and countably additive set functions, regular set functions, outer measures on power set of reals, measurable spaces, Lebesgue measure, uncountable set with measure zero, a non-measurable set, measurable functions and their properties. (Contact hours – 15)

UNIT-II

Lebesgue integrable functions, properties of Lebesgue integrals, Lebesgue's monotone convergence theorem, Fatou's lemma, Lebesgue's dominated convergence theorem; comparison of Riemann and Lebesgue integral; integration of complex valued functions, functions of class \mathcal{L}_2 , Riesz-Fischer theorem. (Contact hours – 15)

UNIT-III

Directional derivatives; derivatives of functions of several variables and their interrelationship; chain rule; mean value theorem; higher order partial derivatives; equality of mixed partial derivatives, Schwarz lemma; Taylor's theorem. (Contact hours – 15)

UNIT-IV

Injective mapping theorem, surjective mapping theorem, inverse function theorem and implicit function theorem of functions of two and three (for analogy) variables; extremum problems with and without constraints of functions of two and three (for analogy) variables. (Contact hours – 15)

Textbooks:

1. Principles of Mathematical Analysis (3rd edition) – W. Rudin, McGraw Hill Kogakusha Ltd., 2017.
2. Mathematical Analysis (5th edition) – T. Apostol, Addison-Wesley; Publishing Company, 2001.

Reference books:

1. The Elements of Real Analysis (3rd edition) – R. G. Bartle, Wiley International Edition, 1994.
2. Advanced Calculus (4th Edition) – R.C. Buck & E.F. Buck, McGraw Hill Book Company, 1999.
3. Introduction to Topology and Modern Analysis (4th edition) – G. F. Simmons, McGraw Hill Kogakusha Ltd., 2000.
4. Introduction to Real Analysis (4th edition) – R. G. Bartle and D. R. Sherbert, John Wiley & Sons, Inc., New York, 2021.

MTH-DSEC-507: TOPOLOGY

[T = Contact hours (Theory) = **60**, P = Contact hours (Practical) = **Nil**, CR = Credits = **4**, Marks=100].

UNIT I

Order relations, dictionary order, well-ordered set, minimal uncountable well ordered set S_Ω ; topological spaces; basis and sub basis; order topology; subspace topology; closed sets and limit points, closure and interior of a set, Hausdorff spaces, continuous functions and homeomorphisms, pasting lemma. (Contact hours–15)

UNIT II

Product topology, quotient topology, connected spaces, path connected spaces, component, path component, locally connected and locally path-connected spaces. (Contact hours – 15)

UNIT III

Compact spaces; limit point compact and sequentially compact spaces; locally compact spaces; one-point compactification; finite product of compact spaces, statement of Tychonoff's theorem and its applications. (Contact hours – 15)

UNIT IV

First and second countable spaces; Lindelof spaces and separable spaces; regular and normal spaces, Urysohn's lemma, completely regular space, Urysohn's metrization theorem and Tietze's extension theorem and applications. (Contact hours – 15)

Textbooks:

1. Topology, a first course – J. R. Munkres, Prentice-Hall of India Ltd., New Delhi, 2000.

Reference books:

1. General Topology – J. L. Kelley, Springer Verlag, New York, 1990.
2. An introduction to general topology (2nd edition) – K. D. Joshi, Wiley Eastern Ltd., New Delhi, 2002.
3. General Topology – J. Dugundji, Universal Book Stall, New Delhi, 1990.
4. Foundations of General Topology – W. J. Pervin, Academic Press, New York, 1964.
5. General Topology – S. Willard, Addison-Wesley Publishing Company, Massachusetts, 1970.
6. Basic Topology – M.A. Armstrong, Springer International Ed., 2005.

MTH-DSEC-508: PARTIAL DIFFERENTIAL EQUATIONS

[T = Contact hours (Theory) = 60, P = Contact hours (Practical) = Nil, CR = Credits = 4, Marks=100].

UNIT I

Meaning of a Partial Differential Equation (PDE), well-posed problems, initial conditions, boundary conditions, first-order PDE in two independent variables and the Cauchy Problem, semilinear and quasilinear equations, method of characteristics, examples of characteristics method, the existence and uniqueness theorem, non-linear PDE of first order, Charpit's method of solution. (Contact hours – 15)

UNIT II

Second order linear PDE, classification, canonical forms of hyperbolic, parabolic and elliptic equations, one-dimensional wave equation, canonical form and general solution, the Cauchy problem and d'Alembert's formula, domain of dependence and region of influence, the Cauchy problem for the non homogeneous wave equation. (Contact hours – 15)

UNIT III

Parabolic differential equations, heat equation, occurrence and importance of heat equation, fundamental solution of the heat equation, separation of variables for the heat equation, uniqueness using energy method for initial boundary value problem, applications of the heat equation. (Contact hours – 15)

UNIT IV

Elliptic equations, basic properties of elliptic problems, Laplace and Poisson equations, the maximum principle, applications of the maximum principle, Green's identities, separation of variables for elliptic problems. (Contact hours – 15)

Text Books:

1. An Introduction to Partial Differential Equations- Yehuda Pinchover and Jacob Rubinstein, Cambridge University Press, 2005.
2. Partial Differential Equations (Second Edition)- Phoolan Prasad and Renuka Ravindran, New Age International (P) limited Publishers, 2011.

Reference Books:

1. Partial Differential Equations- Lawrence C. Evans, American Mathematical Society, Volume 19 of Graduate studies in mathematics, 2010.
2. Lectures on Partial Differential Equations- Vladimir I. Arnold, Springer-Verlag Berlin Heidelberg.
3. Partial Differential Equations: An Introduction – Walter A. Strauss, John Wiley and Sons, Ltd, 2007.

MTH-RM-509: RESEARCH METHODOLOGY & PROPOSAL WRITING

[T = Contact hours (Theory) = 60, P = Contact hours (Practical) = Nil, CR = Credits = 4, Marks=100].

UNIT I

Meaning of research, Objectives of research, Motivation and Types of research, Approaches, Criteria of a good research, research problem, selecting a research problem.
(Contact hours – 15)

UNIT II

Well-ordering property of natural numbers, Principle of Mathematical Induction, Pigeonhole principle, recurrence relations and derangements, Inclusion-exclusion principle.
(Contact hours – 15)

UNIT III

First order differential equation and the dynamical system: the equation $x'(t) = ax$, the logistic population model, constant harvesting bifurcation, periodic harvesting and periodic solution, computing the Poincare map, exploration of a two-parameter family, Computer applications, Introduction to SageMath, Python and Jupyter.
(Contact hours – 15)

UNIT IV

Review of literature: Writing a research proposal using Latex/MSWord. How to create Latex files, How to typeset a Latex document, How to use graphics in Latex, How to setup a beamer document for presentation.
(Contact hours – 15)

Books:

1. C. R. Kothari, Research Methodology – Methods and techniques, New Age International publishers (2004).
2. Robert A. Beeler, How to count, An Introduction to Combinatorics and its applications, Springer (2005).
3. Introduction to Real Analysis (4th edition) – R. G. Bartle and D. R. Sherbert, John Wiley & Sons, Inc., New York, (2021).
4. Differential equations, dynamical systems and introduction to chaos - M. Hirsch, S. Smale & R.L. Devaney; Academic Press, Elsevier, (2013).
5. Paul Zimmerman, Computational Mathematics with Sagemath, Creative Commons (2018).
6. R. Nageswara Rao, Core Python Programming, Third edition, Dreamtech press (2021).

References:

1. Donald Bindner and Martin Erickson, A student's guide to the study, practice and tools of Modern Mathematics, CRC Press (2011).
2. Richard Hammack, Book of Proof, Virginia Commonwealth Univ., Third Edition (2018).

MTH-SEC-510: PROGRAMMING IN C

[T = Contact hours (Theory) = **60**, P = Contact hours (Practical) = **Nil**, CR = Credits = **4**, Marks=100].

UNIT I

Character sets for C; identifiers in C; arithmetic expressions in C; assignment statements in C; built-in functions; input and output statements in C; input and output formatting, Indentation, comment statements; data types; operators – relational, logical, arithmetical and assignment operators. statement labels; elementary programs in C. Logical IF statements in C; break, continue and goto statements in C; problems using if and nested if - Roots (including complex roots) of quadratic equations: checking if a point is inside or outside a circle, checking if a Triangle is isosceles, equilateral, scalene, checking if a year is a leap year. (Contact hours – 15)

UNIT II

Loops- while, for, do-while loops in C. arrays- arrays of numbers – vectors and matrices; reading and writing arrays, operations on arrays; strings, standard string functions, operations on strings. Sum of digits and reversing a number, valuation of sum of series, binomial coefficients. Vectors and Matrices – dot product, angle between vectors, norm1, norm2, norm3, and norm infinity of a vector, finding unit vector along a vector. Input, output, addition, multiplication, trace, transpose for matrices. (Contact hours – 15)

UNIT III

Function definition, function prototypes, arguments, call by value, call by reference, automatic variables in C; scope– local and global variables; recursion vs iteration, file- file opening modes, file input/output using fprintf, fscanf. Recursion – evaluating n!, series, evaluating determinant of a matrix, 8 Queens problem. (Contact hours – 15)

UNIT IV

Number Theory applications– checking if a number is prime , factorising a number into prime factors, Sieve method, Euclidean Algorithm for gcd and Extended Euclidean Algorithm. Programming conjectures like Goldbach conjecture, Collatz conjecture. (Contact hours – 15)

Textbooks:

1. The C Programming Language – B. W. Kernighan and D. M. Ritchie, Prentice Hall, India, 1995.
2. Programming in ANSI C – Balagurusamy, Tata McGraw-Hill, 2008.
3. Primes and Programming – An Introduction to Number Theory with Programming– P. Goblin, Cambridge University Press, 1993.

MTH-CC-600: COMPLEX FUNCTION THEORY

[T = Contact hours (Theory) = 60, P = Contact hours (Practical) = Nil, CR = Credits = 4, Marks=100].

UNIT I

The field of complex numbers, complex form of equations of straight lines, half planes, circles, stereographic projection, completeness property of \mathbb{C} , radius of convergence of power series, analytic/holomorphic function, Cauchy-Riemann equations; analyticity of power series, exponential, cosine and sine, a branch of the logarithm function,

(Contact hours – 15)

UNIT II

Mobius transformation; symmetry and orientation principle; examples of images of regions under elementary analytic function; Integration of complex-valued functions along a piecewise continuously differentiable path, index of a closed curve, Cauchy's theorem and its corollaries.

(Contact hours – 15)

UNIT III

Cauchy's integral formula, Taylor's expansion of holomorphic functions, Cauchy's estimate; Liouville's theorem; fundamental theorem of algebra; zeros of an analytic function and related results; maximum modulus theorem; Schwarz' lemma.

(Contact hours – 15)

UNIT IV

Laurent's expansion of a holomorphic function in an annulus, singularities of a function, removable singularities, poles and essential singularities; residues, calculus of residues; evaluation of definite integrals; argument principle; Rouché's Theorem.

(Contact hours – 15)

Textbooks:

1. Functions of one complex variable – J. B. Conway, Springer International Student edition, Narosa Publishing House, New Delhi, 2000.
2. Elementary Theory of Analytic Functions of one or several complex variables – H. Cartan, Courier Dover Publications, New York, 1995.

Reference books:

1. Complex Analysis (2nd Edition) – L. V. Ahlfors, McGraw-Hill International Student Edition, 1990.
2. Complex Variables and applications – R. V. Churchill, McGraw-Hill, 1996.
3. An Introduction to the Theory of functions of a complex Variable – E. T. Copson, Oxford university press, 1995.
4. An Introduction To Complex Analysis – A. R. Shastri, Macmillan India Ltd., 2003.
5. Complex Variables and Applications – S. Ponnusamy, and H. Silverman, Birkhäuser, 2006.

MTH-CC-601: NUMERICAL ANALYSIS

[T = Contact hours (Theory) = 60, P = Contact hours (Practical) = Nil, CR = Credits = 4, Marks=100].

UNIT I

Numerical solutions of equations. Method of iteration and Newton-Raphson method, Rate of convergence and Algorithms. Solution of system of linear algebraic equations using Gaussian Elimination without and with pivotal condensation- method and Algorithms. Matrix inversion using elementary row operations – Algorithm. Gauss-Seidel method and Algorithm. Matrix norms and Convergence Analysis of Gauss Seidel method. (Contact hours – 15)

UNIT II

Interpolation – Existence and Uniqueness Theorem for polynomial interpolation, Newton's divided Difference and Lagrange's interpolating polynomials. Algorithms and error estimate. Hermite interpolation, Spline interpolation. (Contact hours – 15)

UNIT III

Numerical differentiation and Richardson Extrapolation. Numerical integration based on Interpolation. Error Analysis. Gaussian Quadrature – method, convergence and error analysis. Romberg Integration. (Contact hours – 15)

UNIT IV

Numerical solution of Ordinary differential equations using Euler's method, Picard's method, 2nd order Runge-Kutta methods and Multistep method (Predictor-corrector method). Explicit method, Implicit method and Crank-Nicholson method for solving the one-dimensional heat equation. Stability analysis. Problems without time dependence – Finite difference method. (Contact hours – 15)

Textbooks:

1. Numerical Analysis, David Kincaid, Ward Cheney, Brooks/Cole Publishing Company, Pacific Grove, California, 1991.
2. An Introduction to Numerical Analysis, Second Edition, Kendall E. Atkinson, John Wiley and Sons, 1989.

MTH-CC-602: THEORY OF FIELD EXTENSIONS

[T = Contact hours (Theory) = 60, P = Contact hours (Practical) = Nil, CR = Credits = 4, Marks=100].

UNIT I

Extension fields, finite extensions; algebraic and transcendental elements, adjunction of algebraic elements, Kronecker theorem, algebraic extensions, splitting fields – existence and uniqueness; extension of base field isomorphism to splitting fields.

(Contact hours – 15)

UNIT II

Simple and multiple roots of polynomials, criterion for simple roots, separable and inseparable polynomials; perfect fields; separable and inseparable extensions, finite fields; prime fields and their relation to splitting fields; Frobenius endomorphisms; roots of unity and cyclotomic polynomials.

(Contact hours – 15)

UNIT III

Algebraically closed fields and algebraic closures, primitive element theorem; normal extensions; ruler and compass constructions; automorphism groups and fixed fields; Galois pairing; determination of Galois groups, fundamental theorem of Galois theory, abelian and cyclic extensions.

(Contact hours – 15)

UNIT IV

Normal and subnormal series, composition series, Jordan-Holder theorem (statement only); solvable groups; nilpotent groups; solvability by radicals; solvability of algebraic equations; symmetric functions; fundamental theorem of algebra.

(Contact hours – 15)

Textbooks:

1. Algebra (Second Edition) - Michael Artin, Pearson, 2015.
2. Algebra - Thomas Hungerford, Springer, 2014.

Reference books:

1. Basic Abstract Algebra (South Asia edition) – P. B. Bhattacharya, S. K. Jain and S. R. Nagpal, Cambridge University Press, 2003.
2. Galois Theory – T. I. F. R. Mathematical pamphlets, No. 3, 1965.
3. Topics in Algebra (4th edition) – I. N. Herstein, Wiley Eastern Limited, New Delhi, 2006.
4. A First Course in Abstract Algebra (4th edition) – J. B. Fraleigh, Narosa Publishing House, New Delhi, 2002.
5. Contemporary Abstract Algebra (5th edition) – J. A. Gallian, University of Minnesota, Duluth, 2004.
6. Basic Algebra I (3rd edition) – N. Jacobson, Hindustan Publishing corporation, New Delhi, 2002.
7. Field and Galois Theory – Patrick Morandi, Springer, 1996.

DISCIPLINE-SPECIFIC ELECTIVE COURSES FOR THE THIRD SEMESTER

MTH-DSEC-603: GRAPH THEORY

[T = Contact hours (Theory) = 60, P = Contact hours (Practical) = Nil, CR = Credits = 4, Marks=100].

UNIT I

The Königsberg Bridge problem; definition and examples of a graph; simple graphs; null graph; subgraphs; union of graphs; subdivision of simple graphs; complement of a simple graph; vertex degree and handshaking lemma; adjacency and incidence matrices of a simple graph; graph isomorphism. (Contact hours – 15)

UNIT II

Walks, trails, paths and cycles in a graph; lengths of paths and cycles, girth of a simple graph; distance, diameter, radius and eccentricity in a graph; connected graphs; components of graphs Eulerian and Hamiltonian graphs and their characterizations. (Contact hours – 15)

UNIT III

Regular graphs; bipartite graphs; complete graphs; complete bipartite graphs; characterization of bipartite graphs, trees, spanning trees, minimal spanning trees; cliques and independent sets in a simple graph; vertex coloring. (Contact hours – 15)

UNIT IV

Matching, perfect matching, 1-factorization, planarity of simple graphs; characterization of planar graphs; Euler's formula for planar graphs. (Contact hours – 15)

Textbooks:

1. Introduction to Graph Theory, R. J. Wilson, Addison Wesley Longman Limited, England, 1998.
2. Introduction to Graph Theory, D. B. West, PHI Learning Pvt. Ltd., New Delhi, 2009.

Reference books:

1. Graph Theory, R. Diestel, GTM vol. 173, Springer-Verlag, NY, 2006.
2. A Beginner's Guide to Graph Theory, W.D. Wallis, Birkhäuser Boston, 2007.

MTH-DSEC-604: RING THEORY

[T = Contact hours (Theory) = **60**, P = Contact hours (Practical) = **Nil**, CR = Credits = **4**, Marks=100].

UNIT I

Basic concepts of rings; operations on ideals; matrix rings, polynomial rings; direct products of rings; unit elements in a ring, division rings, fields; idempotent and nilpotent elements in a ring; modules ; sub-modules; direct sums (internal and external) of modules, direct summands, direct products of modules. (Contact hours – 15)

UNIT II

Module homomorphisms, Isomorphism theorems; exact sequences; the group of homomorphisms and its properties relative to exact sequences; free modules; Zorn's lemma, every vector space has a basis; torsion free and torsion modules over commutative domains. (Contact hours – 15)

UNIT III

Projective modules; exact sequences and projectivity; injective modules, injectivity and divisibility over domains; exact sequences and injectivity; Baer's theorem and its elementary applications. (Contact hours –15)

UNIT IV

Simple modules, semisimple modules (as per Bourbaki); Schur's lemma; equivalent conditions for semisimple modules; Wedderburn structure theorem (only statement); characterization of semisimple rings via projective and injective modules. (Contact hours –15)

Textbooks:

1. Algebra, Vol.3: Modules - I. S. Luthar and I.B.S. Passi, Narosa Publishing House, New Delhi, 2002.

Reference books:

1. Elementary Rings and Modules – I. T. Adamson, Oliver and Boyd, Edinburgh, 1995.
2. Notes on Homological Algebra – J. J. Rotman, Van nostrand, 1990.
3. Basic Algebra II (2nd edition) – N. Jacobson, Dover Publications, 2009.
4. Algebra (3rd Edition) – S. Lang, Springer, 2002.

MTH-DSEC-605: ALGEBRAIC TOPOLOGY

[T = Contact hours (Theory) = 60, P = Contact hours (Practical) = Nil, CR = Credits = 4, Marks=100].

UNIT I

Homotopy of maps, homotopy of paths, fundamental group, simply connected spaces, covering spaces, unique lifting theorem, path-lifting theorem, covering homotopy theorem, fundamental group of \mathbb{S}^1 , $\mathbb{S}^1 \times \mathbb{S}^1$; retraction and fixed points; Brouwer's fixed point theorem for 2-disc; fundamental theorem of algebra. (Contact hours – 15)

UNIT II

Homotopy type, contractible space and deformation retract; calculation of fundamental groups of \mathbb{S}^n ($n > 1$) using Van Kampen's theorem (special case); fundamental group of some surfaces, criterion of lifting of maps in terms of fundamental groups; universal coverings and examples. (Contact hours – 15)

UNIT III

Affine preliminaries, chain complex, Singular homology, relation between Π_1 and H_1 ; relative homology, the exact homology sequence (Contact hours – 15)

UNIT IV

Excision theorem (statement only), Calculations of homology of \mathbb{S}^n and its application to Brouwer's fixed point theorem for n-disc ($n > 2$), further applications to spheres and vector fields; Mayer-Vietoris sequence and its applications. (Contact hours – 15)

Textbooks:

1. Topology, a first course – J. R. Munkres, Prentice-Hall of India Ltd., New Delhi, 2000.
2. Algebraic topology, a first course (2nd edition) – M. J. Greenberg and J. R. Harper, Addison-Wesley Publishing co., 1997.

Reference books:

1. Algebraic Topology – A. Hatcher, Cambridge University Press, 2002.
2. Algebraic Topology (2nd edition) – E. H. Spanier, Springer-Verlag, New York, 2000.
3. An Introduction to Algebraic Topology – J. J. Rotman, Graduate Text in Mathematics, No. 119, Springer, New York, 2004.
4. Algebraic topology, a first course (2nd edition) – W. Fulton, Graduate Text in Mathematics, No. 153, Springer, New York, 1995.
5. Foundations of Algebraic Topology (2nd edition) – S. Eilenberg and N. E. Steenrod, Princeton University Press, 1995.

MTH-DSEC-606:THEORY OF RELATIVITY

[T = Contact hours (Theory) = **60**, P = Contact hours (Practical) = **Nil**, CR = Credits = **4**, Marks=100].

UNIT-I

The special theory of relativity: Minkowski space-time, inertial frames of reference, postulates of the special theory of relativity, Lorentz transformations, length contraction, time dilation, variation of mass, composition of velocities, world events, world regions and light cone, equivalence of mass and energy. (Contact hours – 15)

UNIT II

Manifolds and Tensors: Tangent space and vector fields, contravariant and covariant vectors, tensors, manifolds and metric, examples of metrics, covariant derivative and parallel transport, geodesics, properties of geodesics, Riemann curvature tensor, Ricci, Weyl and Einstein tensor, Energy-momentum tensor, properties of curvature tensors. (Contact hours – 15)

UNIT III

General Theory of Relativity: physics in curved space time, principle of covariance, principle of equivalence, Lagrangian formulation, derivation of Einstein's equation, properties of Einstein's equation, Newtonian approximation of Einstein's equations. (Contact hours – 15)

UNIT IV

Solution of Einstein's equation and tests of general relativity: Spherical symmetry and Birkhoff's theorem, the Schwarzschild solution, the Schwarzschild interior solution, geodesics in Schwarzschild space-time, Testing general relativity- bending of light, radar echo delay. (Contact hours – 15)

Textbooks:

1. G. L. Naber, The Geometry of Minkowski Spacetime: An Introduction to the Mathematics of the Special Theory of Relativity, Springer, 2012.
2. Spacetime and Geometry: An Introduction to General Relativity- S. M. Carroll, Cambridge Univ. Press, 2019.
3. Introduction to General Relativity And Cosmology- C. G. Boehmer, World Scientific Publishing Europe Ltd, 2016.

Reference books:

1. Introducing Einstein's Relativity- Ray d'Inverno, Clarendon Press, 1992.
2. A First Course in General Relativity- B. F. Schutz, Cambridge University Press, 2009.
3. General Relativity – R.M. Wald, University of Chicago Press, 1984.

MTH-DSEC-607: FUNCTIONAL ANALYSIS

[T = Contact hours (Theory) = **60**, P = Contact hours (Practical) = **Nil**, CR = Credits = **4**, Marks=100].

UNIT I

Normed linear spaces, Banach spaces; classical Banach spaces, L^p spaces, Holder's inequality, Minkowski's inequality; convergence and completeness, Riesz-Fischer theorem; continuous linear transformations between normed linear spaces; bounded linear functional on L^p spaces, Riesz representation theorem. (Contact hours – 15)

UNIT II

Hahn-Banach theorem and its consequences; embedding of a normed linear space in its second conjugate space; strong and weak topologies; open mapping theorem, closed graph theorem, uniform boundedness theorem, conjugate of an operator. (Contact hours – 15)

UNIT III

Hilbert's space, examples and simple properties; orthogonal complements, orthonormal set, Bessel's inequalities, complete orthonormal sets; Gram-Schmidt orthogonalization process; conjugate space; the adjoint of an operator. (Contact hours – 15)

UNIT IV

Self adjoint operators, normal operators, unitary operators; (orthogonal) projections, spectrum of an operator, spectral theorem for a normal operator on a finite dimensional Hilbert space. (Contact hours – 15)

Textbooks:

1. Real Analysis (4th edition) – H. L. Royden, Macmillan Publishing co. inc, New York, 2011.
2. Introduction to Topology and Modern Analysis (4th edition) – G. F. Simmons, Tata McGraw -Hill Ltd., 2004.

Reference books:

1. Functional Analysis – W. Rudin, Tata McGraw hill Book Company, 2006.
2. Functional Analysis – B. V. Limaye, Wiley Eastern Ltd., 1996.
3. First course in Functional Analysis (2nd edition) – C.Goffman and G. Pedrick, AMS, Chelsea Publishing House, 2017.

MTH-DSEC-608: DIFFERENTIAL GEOMETRY

[T = Contact hours (Theory) = 60, P = Contact hours (Practical) = Nil, CR = Credits = 4, Marks=100].

UNIT I

Vectors; tangent vectors; tangent spaces; tangent vector fields; derivative mappings; translations; affine transformations and rigid motions (isometries); exterior derivatives.

(Contact hours – 15)

UNIT II

Space curves; arc length; tangent vectors and vector fields on a curve; curvature and torsion; Serret-Frenet formulas; osculating plane; osculating circle; osculating sphere; fundamental theorem of local theory of space curves (existence and uniqueness theorems).

(Contact hours – 15)

UNIT III

Surfaces and their (local) parametrization on coordinate systems; change of parameters; parametrized surfaces; curves on surfaces; tangent and normal vectors; tangent and normal vector fields on a surface; first, second and third fundamental forms of a surface at a point; Gauss mapping.

(Contact hours – 15)

UNIT IV

Normal sections and normal curvature of a surface at a point; Meusnier's theorem; elliptic, hyperbolic, parabolic and planar points; Dupin indicatrix; principal directions; principal curvatures of a surface at a point; Mean curvature and Gaussian curvature of a surface at a point; line of curvature; asymptotic curves; conjugate directions; fundamental equations of the local theory of surfaces; statement of Bonnet's fundamental theorem of local theory of surfaces.

(Contact hours – 15)

Textbook:

1. A first course in Differential Geometry – Chun-Chin Hsiung, International Press, 2013.

Reference books:

1. A treatise on the differential geometry of curves and surfaces – P.Eisenhart, Dover Publications, Inc., New York, 2004.
2. Differential Geometry of three dimensions (1st paperback ed.) – C. R. Weatherburn, The English Language Book Society and Cambridge University Press, 2016.
3. An Introduction to differential geometry – T. S. Willmore, Oxford, Clarendon Press, 1979.
4. A course in differential geometry – W. Klingenberg, Graduate Texts in Mathematics 51, Springer-Verlag, 1978.
5. Elementary differential Geometry – A. Pressley, Springer International Edition, 2005.

MTH-DSEC-609: NUMBER THEORY

[T = Contact hours (Theory) = 60, P = Contact hours (Practical) = Nil, CR = Credits = 4, Marks=100].

UNIT I

Divisibility; Euclidean algorithm; primes; congruences; Fermat's theorem, Euler's theorem and Wilson's theorem; Fermat's quotients and their elementary consequences; solutions of congruences; Chinese remainder theorem; Euler's phi-function; congruence modulo powers of prime; power residues; primitive roots and their existence.

(Contact hours – 15)

UNIT II

Quadratic residues; Legendre symbol, Gauss' lemma about Legendre symbol; quadratic reciprocity law; proofs of various formulations; Jacobi symbol; greatest integer function; diophantine equations – solutions of $ax + by = c$, $x^2 + y^2 = z^2$, $x^4 + y^4 = z^2$; properties of Pythagorean triples; sums of two, four and five squares; assorted examples of diophantine equations.

(Contact hours – 15)

UNIT III

Arithmetic functions, multiplicative arithmetic functions (elementary ones); Möbius inversion formula; convolution of arithmetic functions, group properties of arithmetic functions; recurrence functions; Fibonacci numbers and their elementary properties.

(Contact hours – 15)

UNIT IV

Simple continued fractions, finite and infinite continued fractions, uniqueness, representation of rational and irrational numbers as simple continued fractions, rational approximation to irrational numbers, Hurwitz theorem, basic facts of periodic continued fractions and their illustrations; Pell's equation.

(Contact hours – 15)

Textbooks:

1. An Introduction to the Theory of Numbers (5th edition) – I. Niven, H. S. Zuckerman and H. L. Montgomery, Wiley India Pvt. Ltd., 2008.
2. Elementary Number Theory (7th edition) – D. M. Burton, Universal Book Stall, New Delhi, 2011.

Reference books:

1. History of the Theory of Numbers (Vol. II, Diophantine Analysis) – L. E. Dickson, Dover Publications, 2013.
2. An Introduction to the Theory of Numbers (6th edition) – G. H. Hardy and E. M. Wright, The English Language Society and Oxford University Press, 1998.
An Introduction to the Theory of Numbers (3rd edition) – I. Niven and H. S. Zuckerman, Wiley Eastern Ltd., New Delhi, 1993.

MTH-DSEC-610: INTEGRAL EQUATIONS AND CALCULUS OF VARIATIONS

[T = Contact hours (Theory) = **60**, P = Contact hours (Practical) = **Nil**, CR = Credits = **4**, Marks=100].

UNIT I

Definition of integral equations, reduction of ordinary differential equations into integral equations, Fredholm integral equations of first kind with separable kernels, eigen values and eigen functions, method of successive approximation, iterative scheme for Fredholm integral equations of second kind; Volterra integral equations of second kind, resolvent kernel of Volterra equation and its results, application of iterative scheme to Volterra equation of the second kind, convolution type kernels.

(Contact hours – 15)

UNIT II

Fourier Integral Transform, properties of Fourier Transform, Fourier sine and cosine transforms, application of Fourier transform to ordinary and partial differential equations of initial and boundary value problems, evaluation of definite integrals; basic properties of Laplace Transform, convolution theorem and properties of convolution, inverse Laplace transform, application of Laplace transform to solution of ordinary and partial differential equations of initial and boundary value problems, evaluation of definite integrals the inversion theorem, evaluation of inverse transforms by residue method.

(Contact hours – 15)

UNIT III

Calculus of variation with one independent variable: Function spaces, functionals, variations, Euler's equation with fixed boundary of the functional containing only the first order derivative of the only dependent variable with respect to one independent variable, variational problems with functional having higher order derivatives of the only dependent variable, general case of Euler's equation, applications.

(Contact hours – 15)

UNIT IV

Calculus of Variation with several independent variables: Variational problems with functional dependent on functions of several independent variables having first order derivatives, variational problems in parametric form, variational problems with subsidiary condition: Isoperimetric problems, applications.

(Contact hours – 15)

Text Books:

1. R. P. Kanwal, Linear Integral Equations, Theory and Techniques, Academic Press, New York, 1971.
2. M. R. Spiegel, Schaum's Outline Series: Theory and Problems of Laplace Transforms, McGraw- Hill Book Company, 1965.
3. A. S. Gupta, Calculus of variation with Applications : Prentice Hall of India, 1999.

Reference Books:

1. S. G. Mikhlin, Linear Integral Equations (Translated from Russia), Hindustan Book Agency, 1960.
2. I. M. Gelfand, and A. S. Richard. Calculus of variations, Dover Publications, 2000.

3. F. B. Hilderbrand, Methods of Applied Mathematics, Dover Publications, 1992.
4. R. Courant and D. Hilbert, Methods of Mathematical Physics- Vol- I, Wiley Interscience, New York 1953.

MTH-DSEC-611: RIEMANNIAN GEOMETRY

[**T** = Contact hours (Theory) = **60**, **P** = Contact hours (Practical) = **Nil**, **CR** = Credits = **4**, Marks=100].

UNIT I

Topological manifolds, smooth structures, smooth manifold chart lemma, smooth functions and smooth maps between smooth manifolds, diffeomorphisms.

(Contact hours – 15)

UNIT II

Tangent vectors on smooth manifolds, differential of a smooth map, the tangent bundle, vector fields, tensors, tensor bundles and tensor fields on smooth manifolds.

(Contact hours – 15)

UNIT III

Riemannian metric on smooth manifolds, isometries, length and distances, affine connections, covariant derivatives, geodesics, parallel transport. (Contact hours – 15)

UNIT IV

Levi-Civita connection, Christoffel symbols, curvature tensor, symmetries of curvature tensor, Ricci and scalar curvatures, Einstein metric. (Contact hours – 15)

Textbooks:

1. Introductions to smooth manifolds (2nd edition)– John M. Lee, Springer Science+Business Media New York, 2013.
2. Introductions to Riemannian manifolds (2nd edition)– John M. Lee, Springer International Publishing AG, 2018.

Reference books:

1. Introductions to manifolds (2nd edition)– Loring, W. Tu , Springer Science+ Business Media, LLC, 2011.
2. Manifolds and differential geometry- Jeffery M. Lee, American Mathematical Society , 2009.
3. Differential geometry and topology: With a view to dynamical system- Keith Burns and Marian Gidea, Chapman & Hall/CRC, 2005.