

SAMPLE Questions

DEPARTMENT OF STATISTICS
NORTH EASTERN HILL UNIVERSITY
Ph.D./M.Phil. ENTRANCE TEST

Full Marks: 25

Time: 2 Hrs

Date: 2018 Feb 2020

Figures in the margin indicate marks for the questions.

UNIT I

(Marks: 10)

Answer any FIVE questions from Unit I

1. Consider a simple linear regression model $y = \beta_0 + \beta_1 x + \varepsilon$ with uncorrelated errors having mean 0 and variance σ^2 . b_0 and b_1 are the estimates of β_0 and β_1 respectively, then obtain $E(b_0)$, $E(b_1)$, $\text{Var}(b_0)$ and $\text{Var}(b_1)$. (2)
2. An experiment consists of casting an unbiased die twice. Define the events,
 $E_i = \{\text{The face value on the first casting is } i\}$, $i = 1, 2, \dots, 6$.
 $F_j = \{\text{the sum of face values is } j\}$, $j = 2, 3, \dots, 12$.
Which pair(s) of $\{E_i, F_j\}$ are independent? (2)
3. An urn contains $2m$ balls of two different colors in equal proportion. If balls are drawn one by one (a) without replacement, (b) with replacement – compute probability that the color will alternate. (2)
4. You have to test a hypothesis regarding a population location on the basis of a random sample of size n . Write down your considerations towards performance of the test. (2)
5. Obtain the layout of a 3^3 factorial experiment having 3 replicates with 3 blocks per replicate such that ABC, AB²C and ABC² are confounded in different replicates. (2)
6. Consider a simple linear regression model $y = \beta_0 + \beta_1 x + \varepsilon$ with uncorrelated errors having mean 0 and variance σ^2 . How would you test the significance of regression using analysis of variance approach? (2)
7. Discuss briefly how to obtain simultaneous confidence intervals for differences between treatment means. How these are different from one-at-a-time confidence intervals? (2)
8. Write in brief on exponential family of distributions and its role in statistical inference. (2)
9. Show that m.g.f. of binomial distribution tends to Poisson distribution. (2)
10. If $X \sim \beta_1(m, n)$, find the distribution of $Y = \frac{1}{X}$. (2)

UNIT II

(Marks: 15)

Answer any FIVE questions from Unit II

11. What is complete life table? Discuss in brief about its various functions. (3)
12. Let X be a $(p \times 1)$ vector with random variables $E(X) = \mu_{(p \times 1)}$ and Σ = variance-covariance matrix of X . Prove that $E(XX')$ = $\Sigma + \mu\mu'$. (3)
13. What is generalized least squares (GLS)? Obtain GLS estimator of parameter vector, β in brief. (3)
14. If $X \sim N_2(\mu, \Sigma)$, then obtain the distribution of $X_1 - 2X_2$ (3)
15. Name kinds of nuptiality tables? Explain any one of them in brief. (3)
16. (a) Discuss about ridge regression.
(b) What are principal components? Explain principal components regression in brief. (1 + 2 = 3)

17. Let $\{\varepsilon_t\}$ be a stationary process with mean zero and let 'a' and 'b' be constants. If

$X_t = a + bt + s_t + \varepsilon_t$, where s_t is a seasonal component with period 12, show that

$$\nabla \nabla^{12} X_t = (1 - B)(1 - B^{12})X_t$$

is stationary and express its autocovariance function in terms of that of $\{\varepsilon_t\}$. (3)

18. Let $\{X_t\}$ be the AR(1) process defined as (3)

$$X_t = \phi X_{t-1} + \varepsilon_t$$

and $\{\varepsilon_t\} \sim WN(0, \sigma^2)$ and $|\phi| < 1$.

Compute the variance of the sample mean $\frac{X_1 + X_2 + X_3 + X_4}{4}$, when $\phi = 0.9$ and $\sigma^2 = 1$.

Repeat (i) when $\phi = -0.9$ and compare your answer with the result obtained in (i).

19. Express sample mean in terms of population units for SRSWR. Hence or otherwise show that the sample means is an unbiased estimator for population mean. Also find its sampling variance. (3)

20. Describe PPSWR sampling and suggest an unbiased estimator of population total showing its unbiasedness. Also derive its sampling variance. (3)

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